

Chapter 9.4-9.5 Practice Test

Pre-Calculus | Sequences and Series

Name _____
 Date _____
 Period _____
 Grade _____

- I. Find the first four terms of the given sequence and state whether it is arithmetic, geometric or neither.

a. $t_n = n^3 + 1$

$$t_1 = (1)^3 + 1 = 2$$

$$t_2 = (2)^3 + 1 = 9$$

$$t_3 = (3)^3 + 1 = 28$$

$$t_4 = (4)^3 + 1 = 65$$

Neither arithmetic
or geometric

b. $t_n = 16(2)^{2n}$

$$t_1 = 16(2)^{2(1)} = 64$$

$$t_2 = 16(2)^{2(2)} = 256$$

$$t_3 = 16(2)^{2(3)} = 1024$$

$$t_4 = 16(2)^{2(4)} = 4096$$

Geometric

c. $t_n = 4n - 3$

$$t_1 = 4(1) - 3 = 1$$

$$t_2 = 4(2) - 3 = 5$$

$$t_3 = 4(3) - 3 = 9$$

$$t_4 = 4(4) - 3 = 13$$

Arithmetic

d. $t_n = \frac{n-1}{2n+3}$

$$t_1 = \frac{1-1}{2(1)+3} = 0$$

$$t_2 = \frac{2-1}{2(2)+3} = \frac{1}{7}$$

$$t_3 = \frac{3-1}{2(3)+3} = \frac{2}{9}$$

$$t_4 = \frac{4-1}{2(4)+3} = \frac{3}{11}$$

- II. Find the recursive and explicit rules for the following arithmetic or geometric sequences and find the 50th term.

a. 8, 6, 4, 2, ...

R [Next = Now - 2
Starting @ 10] $\xrightarrow{\text{OR}}$ $a_n = a_{n-1} - 2$
 $a_1 = 8$
E [$a_n = -2n + 10$] $\xrightarrow{a_n = -2(n-1) + 8}$
 $a_{50} = -2(50) + 10 = \boxed{-90}$

b. 24, -12, 6, -3, ...

R [Next = Now $(-\frac{1}{2})$
Starting @ -48] $\xrightarrow{\text{OR}}$ $a_n = a_{n-1} (-\frac{1}{2})$
 $a_1 = 24$
E [$a_n = (-48)(-\frac{1}{2})^n$] $\xrightarrow{a_n = (24)(-\frac{1}{2})^{n-1}}$
 $a_{50} = (-48)(-\frac{1}{2})^{50} = \boxed{-4.263 \times 10^{-14}}$

c. -5, -1, 3, 7, ...

d. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, \dots$

R [Next = Now + 4
Starting @ -9] $\xrightarrow{\text{OR}}$ $a_n = a_{n-1} + 4$
 $a_1 = -5$
E [$a_n = 4n - 9$] $\xrightarrow{a_n = 4(n-1) - 5}$
 $a_{50} = 4(50) - 9 = \boxed{191}$

R [Next = Now (2)
Starting @ $\frac{1}{16}$] $\xrightarrow{\text{OR}}$ $a_n = a_{n-1} (2)$
 $a_1 = \frac{1}{8}$
E [$a_n = (\frac{1}{16})(2)^n$] $\xrightarrow{a_n = (\frac{1}{8})(2)^{n-1}}$
 $a_{50} = (\frac{1}{16})(2)^{50} = \boxed{7.037 \times 10^{13}}$

III. How many terms are in the arithmetic sequence 18, 24, ..., 336? What is the 10th term?

Explicit Rule Let!

$$a_n = 6n + 12$$

$$336 = 6n + 12$$

$$a_{10} = 6(10) + 12$$

$$\begin{cases} n = 54 \\ a_1 = 72 \end{cases}$$

IV. Find the indicated term of the sequence.

a. $t_8 = 25, t_{20} = 61, t_2 = ?$ (arithmetic)

$$\begin{matrix} n & a_n \\ (8, 25) & \\ (20, 61) & \end{matrix} \quad m = \frac{61 - 25}{20 - 8} = \frac{36}{12} = 3$$

$$a_n = 3n + a_0$$

$$\begin{matrix} 25 = 3(8) + a_0 \\ 25 = 24 + a_0 \\ -24 \quad -24 \end{matrix}$$

So...

$$a_n = 3n + 1$$

$$a_2 = 3(2) + 1 = 7$$

$$-192 = (a_0)(r)^4 \rightarrow \frac{-192}{r^4} = a_0$$

$$196,608 = (a_0)(r)^9 \rightarrow \frac{196,608}{r^9} = a_0$$

$$196,608 = \left(\frac{-192}{r^4}\right)(r^8)^5 \quad a_0 = \frac{-192}{(-4)^4} = -\frac{3}{4}$$

$$196,608 = -192 r^5 \quad a_n = \left(-\frac{3}{4}\right)(-4)^n$$

$$\sqrt[5]{-192} = \sqrt[5]{r^5}$$

$$-4 = r$$

$$a_7 = \left(-\frac{3}{4}\right)(-4)^7 = 12,288$$

b. $t_2 = 64, t_5 = -8, t_8 = ?$ (geometric)

$$\begin{matrix} n & a_n \\ (2, 64) & \\ (5, -8) & \end{matrix} \quad a_n = (a_0)(r)^n$$

$$64 = (a_0)(r)^2 \rightarrow \frac{64}{r^2} = a_0$$

$$-8 = (a_0)(r)^5$$

$$-8 = \left(\frac{64}{r^2}\right)(r^5)^3 \rightarrow \left(-\frac{1}{8}\right)^3 = \sqrt[3]{r^3}$$

$$-8 = \frac{64 r^3}{64} \rightarrow -\frac{1}{2} = r$$

$$-\frac{1}{8} = r^3 \quad \frac{64}{(-\frac{1}{2})^2} = \frac{64}{\frac{1}{4}}$$

$$a_0 = 256$$

$$a_n = 256 \left(-\frac{1}{2}\right)^n$$

d. $t_3 = 14, t_8 = -3.5, t_{10} = ?$ (arithmetic)

$$\begin{matrix} (3, 14) & \\ (8, -3.5) & \end{matrix} \quad m = \frac{-3.5 - 14}{8 - 3} = \frac{-17.5}{5} = -\frac{7}{2}$$

$$a_n = -\frac{7}{2}n + a_0 \quad a_n = -\frac{7}{2}n + \frac{49}{2}$$

$$14 = -\frac{7}{2}(3) + a_0$$

$$14 = -\frac{21}{2} + a_0$$

$$+\frac{21}{2} + \frac{21}{2}$$

$$\frac{49}{2} = a_0$$

$$a_{10} = -\frac{21}{2}$$

$$3(10) + 2 = 122$$

V. Find the sum of the arithmetic series and make sure to write out the summation notation.

a. $S_{200} : t_1 = 18, t_{200} = 472$

$$\begin{matrix} \text{Means Sum} \\ \text{of 200 terms} \end{matrix}$$

$$m = \frac{472 - 18}{200 - 1} = \frac{454}{199}$$

$$18 = \left(\frac{454}{199}\right) + a_0$$

$$\sum_{k=1}^{200} \left(\frac{454}{199}k + \frac{3128}{199} \right) = \frac{200(18+472)}{2}$$

W H O O H !

$$= 49,000$$

b. $S_{40} : t_1 = 5, t_3 = 11$

$$m = \frac{11 - 5}{3 - 1} = \frac{6}{2} = 3 \quad \sum_{k=1}^{40} (3k + 2) = \frac{40(5+122)}{2}$$

$$5 = 3(1) + a_0$$

$$-3 = -3$$

$$2 = a_0$$

$$+\frac{21}{2} + \frac{21}{2}$$

$$\frac{49}{2} = a_0$$

$$= 2540$$

c. $-8 - 3 + 2 + \dots + 92$

5 5

$$a_n = 5n - 13$$

$$92 = 5n - 13 + 13$$

$$\frac{105}{5} = \frac{5n}{5}$$

$$21 = n$$

$$\sum_{k=1}^{21} 5n - 13 = \frac{21(-8+92)}{2}$$

$$= 882$$

$$\sum_{k=1}^8 3n - 14 = \frac{8(-11+10)}{2} = \frac{-8}{2} = -4$$

VI. Tell whether the described or given sequence converges or diverges. If it converges, tell at what value it converges to.

a. $\lim_{n \rightarrow \infty} \frac{n^2+1}{n^2}$

$$\frac{\infty^2 + 1}{\infty^2} = 1$$

converges to 1

b. $\lim_{n \rightarrow \infty} \frac{4n-3}{2n+1}$

$$\frac{4(\infty) - 3}{2(\infty) + 1} = 2$$

converges to 2

c. $\lim_{n \rightarrow \infty} \frac{n^2-1}{n+1}$

$$\frac{(\infty)^2 - 1}{(\infty) + 1} = \frac{\infty^2}{\infty} = \infty$$

diverges

d. $\lim_{n \rightarrow \infty} \frac{n-1}{n^2}$

$$\frac{\infty - 1}{(\infty)^2} = \frac{\infty}{\infty^2} = \frac{1}{\infty} = 0$$

converges to 0

e. $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)$

$$1 - \left(\frac{1}{\infty}\right) = \\ 1 - 0 = 1$$

converges to 1

f. $\lim_{n \rightarrow \infty} \frac{5n}{n^{1/2}-3}$

$$\frac{5(\infty)}{(\infty)^{\frac{1}{2}} - 3} = \frac{5\infty}{\sqrt{\infty} - 3} \quad \begin{matrix} \text{getting bigger} \\ \text{faster} \end{matrix}$$

getting bigger slower

diverges

VII. Determine whether the geometric series converges. If it does, find its sum.

a. $\sum_{k=1}^{\infty} 2 \left(\frac{3}{4}\right)^k$

$$\sum_{k=1}^{\infty} 2 \left(\frac{3}{4}\right)^k = \frac{\frac{3}{2} \left(1 - \left(\frac{3}{4}\right)^{\infty}\right)}{1 - \left(\frac{3}{4}\right)} = \frac{\frac{3}{2}}{\frac{1}{4}} = \boxed{6}$$

Converges to

b. $\sum_{k=1}^{\infty} 2 \left(-\frac{4}{3}\right)^k$

diverges

c. $\sum_{k=1}^{\infty} 3 \left(\frac{1}{2}\right)^k$

$$\sum_{k=1}^{\infty} 3 \left(\frac{1}{2}\right)^k = \frac{\frac{3}{2} \left(1 - \left(\frac{1}{2}\right)^{\infty}\right)}{1 - \left(\frac{1}{2}\right)} = \frac{\frac{3}{2}}{\frac{1}{2}} = \boxed{3}$$

Converges

d. $\sum_{k=1}^{\infty} 2 \left(-\frac{1}{3}\right)^k$

$$\sum_{k=1}^{\infty} 2 \left(-\frac{1}{3}\right)^k = \frac{-\frac{2}{3} \left(1 - \left(-\frac{1}{3}\right)^{\infty}\right)}{1 - \left(-\frac{1}{3}\right)} = \frac{-\frac{2}{3}}{\frac{4}{3}} = \boxed{-\frac{1}{2}}$$

Converges to

VIII. Find the sum of the geometric sequence and write the summation notation...

a. $-3, -1, -\frac{1}{3}, -\frac{1}{9}, -\frac{1}{27}, \dots$

$$\sum_{k=1}^{\infty} (-9) \left(\frac{1}{3}\right)^k = \frac{-3 \left(1 - \left(\frac{1}{3}\right)^{\infty}\right)}{1 - \frac{1}{3}} = \frac{-3}{\frac{2}{3}}$$

$$= -3 \cdot \frac{3}{2} \boxed{\frac{-9}{4}}$$

b. $1, -2, 4, -8, \dots, -8192$

$$\sum_{k=1}^{14} \left(-\frac{1}{2}\right)(-2)^k = \frac{1 \left(1 - (-2)^{14}\right)}{1 - (-2)} = \boxed{14 = k}$$

$$\boxed{-5461}$$

$$(-8192) = \left[\left(-\frac{1}{2}\right)(-2)^k \right]_{(-2)}$$

$$\log(16384) = \log(-2^k)$$

