Practice Test: 2.6-2.8

Pre-Calculus | Mr. Hurni

(Non-Calculator)

Name:_____

Date:

1. Describe how the graph of the given function can be obtained by transforming the graph of the reciprocal function $f(x) = \frac{1}{x}$. Identify the horizontal and vertical asymptotes, describing using limit notation. Graph the function.

a. $g(x) = \frac{-x+7}{x-5}$ ← Not transformation form

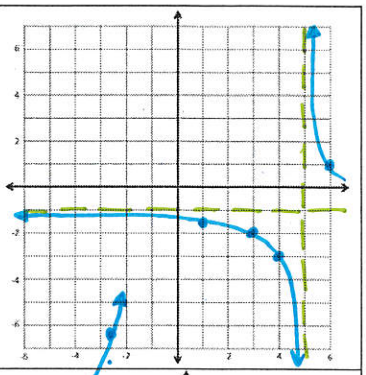
$$x-5\overline{)-x+7}\quad \begin{array}{r} -1 \\ +x\mp 5 \\ \hline 2 \end{array}$$

$= -1 + \frac{2}{x-5}$

OR

$y = \frac{2}{x-5} - 1$

- Vertical stretch by a factor of 2
- Shift right 5 units
- Vertical shift down 1 units



b. $P(x) = \frac{4x+3}{x+2}$

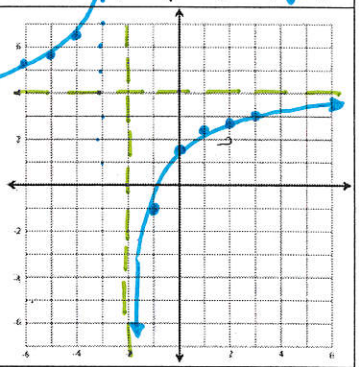
$$x+2\overline{)4x+3}\quad \begin{array}{r} 4 \\ -4x\mp 8 \\ \hline -5 \end{array}$$

$= 4 + \frac{-5}{x+2}$

OR

$y = \frac{-5}{x+2} + 4$

- Vertical reflection
- Vertical stretch by a factor of 5
- Shift left 2 units
- Shift up 4 units



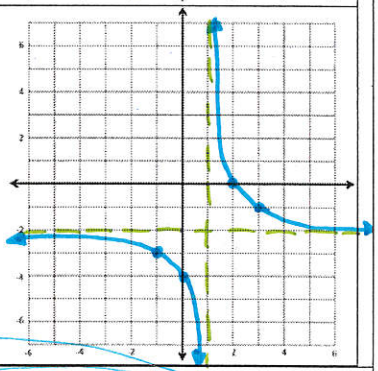
c. $h(x) = \frac{-2x+4}{x-1}$

$$x-1\overline{)-2x+4}\quad \begin{array}{r} -2 \\ +2x\mp 2 \\ \hline 2 \end{array}$$

$= -2 + \frac{2}{x-1}$

$y = \frac{2}{x-1} - 2$

- Vertical stretch by a factor of 2
- Shift down 2 units
- Shift right 1 unit



2. Solve the equation:

a. $\frac{x}{x+2} + \frac{5}{x-3} = \frac{25}{x^2-x-6}$ $(x-3)(x+2)$

$(x-3)(x+2)$

$x(x-3) + 5(x+2) = 25$

$x^2 - 3x + 5x + 10 = 25$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x = -5$ ~~$x = 3$~~ extraneous

b. $\frac{2x}{x+3} + \frac{3}{x-1} = \frac{16}{x^2+2x-3}$ $(x+3)(x-1)$

$(x+3)(x-1)$

$2x(x-1) + 3(x+3) = 16$

$2x^2 - 2x + 3x + 9 = 16$

$2x^2 + x - 7 = 0$

~~$(2x\ )(x\ ) = 0$~~

$x = \frac{-1 \pm \sqrt{1-4(2)(-7)}}{4}$

$x = \frac{-1 \pm \sqrt{57}}{4}$

c. $3t + \frac{5}{t} = 12$ t

$3t^2 + 5 = 12t$

$3t^2 - 12t + 5 = 0$

~~$(3t\ )(t\ 5) = 0$~~

$x = \frac{12 \pm \sqrt{144-4(3)(5)}}{6} = \frac{12 \pm \sqrt{84}}{6} = \frac{12 \pm \sqrt{4}\sqrt{21}}{6}$

$= \frac{12 \pm 2\sqrt{21}}{6}$

$= 2 \pm \frac{\sqrt{21}}{3}$

3. Factor and solve the inequality using a sign analysis chart.

a. $3x^4 + x^3 - 36x^2 + 36x + 16 \geq 0$; a zero occurs at $x = -4$ and $x=2$

$$\begin{array}{c|ccccc} -4 & 3 & 1 & -36 & 36 & 16 \\ & \downarrow & -12 & 44 & -32 & -16 \\ \hline & 3 & -11 & 8 & 4 & 0 \\ 2 & \downarrow & 6 & -10 & -4 & \\ \hline & 3 & -5 & -2 & 0 & \end{array}$$

$f(x) = (x+4)(x-2)(3x^2-5x-2)$

$f(x) = (x+4)(x-2)(3x+1)(x-2)$

$f(x) = (x+4)(3x+1)(x-2)^2$

$x = -4 \quad x = -\frac{1}{3} \quad x = 2$ w/multiplicity 2

$(-)(-)(-)^2$ zero $(+)(-)(-)^2$ zero $(+)(+)(-)^2$ zero $(+)(+)(+)^2$

pos -4 Neg $-\frac{1}{3}$ pos 2 pos

$f(x) \geq 0$ on $(-\infty, -4] \cup [-\frac{1}{3}, \infty)$

b. $6t^3 - 19t^2 + 12t + 3 > t - 3$; a zero occurs at $t = -\frac{1}{3}$

$-t+3 \quad -t+3$

$6t^3 - 19t^2 + 11t + 6 > 0$

$$\begin{array}{c|ccc} -\frac{1}{3} & 6 & -19 & 11 & 6 \\ & \downarrow & -2 & 7 & -6 \\ \hline & 6 & -21 & 18 & 0 \end{array}$$

$f(x) = (3x+1)(6x^2-21x+18)$

$f(x) = (3x+1)(3)(2x^2-7x+6)$

$f(x) = (3x+1)(3)(2x-3)(x-2)$

$x = -\frac{1}{3} \quad x = \frac{3}{2} \quad x = 2$

$(-)(-)(-)$ zero $(+)(-)(-)$ zero $(+)(+)(-)$ zero $(+)(+)(+)$

Neg $-\frac{1}{3}$ pos $\frac{3}{2}$ Neg 2 pos

$f(x) > 0$ on $(-\frac{1}{3}, \frac{3}{2}) \cup (2, \infty)$

c. $2r^3 - 3r^2 - 11r + 6 \leq 0$; a zero occurs at $r = 3$

$$\begin{array}{c|ccc} 3 & 2 & -3 & -11 & 6 \\ & \downarrow & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

$f(x) = (x-3)(2x^2+3x-2)$

$f(x) = (x-3)(2x-1)(x+2)$

$x = 3 \quad x = \frac{1}{2} \quad x = -2$

$(-)(-)(-)$ zero $(-)(-)(+)$ zero $(-)(+)(+)$ zero $(+)(+)(+)$

Neg -2 pos $\frac{1}{2}$ Neg 3 pos

$f(x) \leq 0$ on $(-\infty, -2] \cup [\frac{1}{2}, 3]$

CALCULATOR SECTION (OK to use calculator)

4. Analyze the following graphs.

a. $h(x) = \frac{x^3+x^2-2x+5}{x^3+2}$

$x^3+x^2-2x+5=0$ calculator

-2 5

This one isn't great review for test problems but it still gets the point across.

x-intercepts: $x = -2.552$

y-intercept: $(0, \frac{5}{2})$

Domain: $(-\infty, \sqrt[3]{-2}) \cup (\sqrt[3]{-2}, \infty)$

~~Range~~: $(-\infty, 1) \cup (1, \infty)$ Not on test

Vertical Asymptotes (Limit Notation):

$x^3+2=0 \quad \sqrt[3]{x^3}=\sqrt[3]{-2} \quad x=\sqrt[3]{-2}$

Horizontal Asymptote (Limit Notation): degree of numerator = degree of denominator $y = \frac{a_1}{a_2}$

H.A. @ $y=1$

~~Bounded~~: No Not on test

$\lim_{x\to\sqrt[3]{-2}^-} h(x) = -\infty$

$\lim_{x\to\sqrt[3]{-2}^+} h(x) = \infty$

~~Max/Min~~: Not on test

$\lim_{x\to\infty} h(x) = 1$

$\lim_{x\to-\infty} h(x) = 1$

b. $g(x) = \frac{x-1}{x^2-x-6}$

x-intercepts: $x-1=0 \quad x=1 \quad (1,0)$

y-intercept: $(0, \frac{1}{6})$

Domain: $(-\infty,-2) \cup (-2,3) \cup (3,\infty)$

~~Range~~: $(-\infty, \infty)$

Vertical Asymptotes (Limit Notation):

$x^2-x-6=0$

$(x-3)(x+2)=0$

~~Bounded~~: No

V.A. @ $x=3$ $x=-2$

$\lim_{x\to-2^-} g(x) = -\infty$ $\lim_{x\to-2^+} g(x) = \infty$

$\lim_{x\to3^-} g(x) = -\infty$ $\lim_{x\to3^+} g(x) = \infty$

Horizontal Asymptote (Limit Notation): degree of numerator is less than degree of denominator.

H.A. @ $y=0$

~~Max/Min~~: No

$\lim_{x\to-\infty} g(x) = 0$

$\lim_{x\to\infty} g(x) = 0$

c. $h(x) = \frac{2x+1}{x^2-4x+3}$

x-intercepts: $2x+1=0 \quad x=-\frac{1}{2} \quad (-\frac{1}{2}, 0)$

y-intercept: $(0, \frac{1}{3})$

Domain: $(-\infty,1) \cup (1,3) \cup (3,\infty)$

~~Range~~: $(-\infty, -4.791] \cup [-.208, \infty)$ used calculator

Vertical Asymptotes (Limit Notation):

$x^2-4x+3=0$

$(x-3)(x-1)=0$

~~Bounded~~: No

V.A. @ $x=3$ $x=1$

$\lim_{x\to1^-} h(x) = \infty$ $\lim_{x\to1^+} h(x) = -\infty$

$\lim_{x\to3^-} h(x) = -\infty$ $\lim_{x\to3^+} h(x) = \infty$

Horizontal Asymptote (Limit Notation):

H.A. @ $y=0$

~~Max/Min~~: (1.791, -4.791) (-2.791, -.208) used calculator

$\lim_{x\to-\infty} h(x) = 0$

$\lim_{x\to\infty} h(x) = 0$

d. $g(x) = \frac{x-1}{x^2-x-6}$ Same as b

x-intercepts:

y-intercept:

Domain:

Range:

Vertical Asymptotes (Limit Notation):

Horizontal Asymptote (Limit Notation):

Bounded:

Max/Min:

e. $h(t) = \frac{t-1}{t^2-2t-8}$

x-intercepts: $t-1=0$, $t=1$, $(1,0)$

y-intercept: $(0, \frac{1}{8})$

Domain: $(-\infty,-2) \cup (-2,4) \cup (4,\infty)$

~~Range~~: $(-\infty,\infty)$

Vertical Asymptotes (Limit Notation):

$t^2-2t-8=0$

$(t-4)(t+2)=0$

V.A. @ $t=4$, $t=-2$

$\lim_{x\to -2^-} h(t) = -\infty$ $\lim_{x\to -2^+} h(t) = \infty$ $\lim_{x\to 4^-} h(t) = -\infty$ $\lim_{x\to 4^+} h(t) = \infty$

Horizontal Asymptote (Limit Notation):

H.A. @ $y=0$

$\lim_{x\to -\infty} h(t) = 0$

$\lim_{x\to \infty} h(t) = 0$

~~Bounded~~: No

~~Max/Min~~: No

f. $g(x) = \frac{3x^2+1}{x^2+5x+6}$

$3x^2+1=0$

$-1 \quad -1$

$\frac{3x^2}{3} = \frac{-1}{3}$

$\sqrt{x^2} = \sqrt{-\frac{1}{3}}$

x-intercepts: No x-intercepts

y-intercept: $(0, \frac{1}{6})$

Domain: $(-\infty,-3) \cup (-3,-2) \cup (-2,\infty)$

~~Range~~: $(-\infty,-76.157] \cup [.158,\infty)$

Vertical Asymptotes (Limit Notation):

$x^2+5x+6=0$

$(x+3)(x+2)=0$

V.A. @ $x=-3$, $x=-2$

$\lim_{x\to -3^-} g(x) = \infty$ $\lim_{x\to -3^+} g(x) = -\infty$ $\lim_{x\to -2^-} g(x) = -\infty$ $\lim_{x\to -2^+} g(x) = \infty$

Horizontal Asymptote (Limit Notation):

H.A. @ $y=3$

$\lim_{x\to -\infty} g(x) = 3$ $\lim_{x\to \infty} g(x) = 3$

~~Bounded~~:

~~Max/Min~~: $(-2.405,-76.157)$, $(.139,.158)$

5. Factor and solve the rational inequality using a sign analysis chart.

a. $\frac{x^2+3x-10}{x^2-6x+9} < 0$

x-int

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x=-5 \quad x=2$

Discontinuities

$x^2-6x+9=0$

$(x-3)(x-3)=0$

V.A. @ $x=3$

$\frac{(-)(-)}{(-)(-)}$ | zero | $\frac{(+)(-)}{(+)(-)}$ | zero | $\frac{(+)(+)}{(-)(-)}$ | V.A. | $\frac{(+)(+)}{(+)(+)}$

Pos -5 Neg 2 Pos 3 Pos

$f(x)=\frac{(x+5)(x-2)}{(x-3)(x-3)}$

$f(x)<0$ on $(-5,2)$

Just for fun $f(x) \overset{?}{\geq} 0$ on $(-\infty,-5] \cup [2,3)$

b. $\frac{2a-5}{a^2-9} < 0$

x-int

$2a-5=0$

$\frac{2a}{2}=\frac{5}{2}$

$a=\frac{5}{2}$

Disc.

$a^2-9=0$

$(a+3)(a-3)=0$

$a=-3 \quad a=3$

$\frac{(-)}{(-)(-)}$ | V.A. | $\frac{(-)}{(+)(-)}$ | zero | $\frac{(+)}{(+)(-)}$ | V.A. | $\frac{(+)}{(+)(+)}$

Neg -3 pos $\frac{5}{2}$ Neg 3 pos

$f(a)=\frac{(2a-5)}{(a+3)(a-3)}$

$f(a)<0$ on $(-\infty,-3) \cup (\frac{5}{2},3)$

Just for fun $f(a) \overset{?}{\geq} 0$ on $(-3,\frac{5}{2}] \cup (3,\infty)$

c. $\frac{2x^2-x-1}{x^2-3x+2} \geq 0$

x-int

$2x^2-x-1=0$

$(2x+1)(x-1)=0$

$x=-\frac{1}{2} \quad$ ~~$x=1$~~

Disc.

$x^2-3x+2=0$

$(x-2)(x-1)$

$x=2 \quad x=1$ P.D.

$\frac{(-)}{(-)}$ | zero | $\frac{(+)}{(-)}$ | P.D. | $\frac{(+)}{(-)}$ | V.A. | $\frac{(+)}{(+)}$

pos $-\frac{1}{2}$ neg 1 neg 2 pos

$f(x)=\frac{(2x+1)(x-1)}{(x-2)(x-1)}=\frac{2x+1}{x-2}$

$f(x) \geq 0$ on $(-\infty,-\frac{1}{2}] \cup (2,\infty)$

Just for fun $f(x) \overset{?}{\leq} 0$ on $[-\frac{1}{2},1) \cup (1,2)$

6. Solve the equation.

a. $\frac{3x}{x+2}+\frac{2}{x-1}=\frac{5}{x^2+x-2}$ $(x+2)(x-1)$

$3x(x-1)+2(x+2)=5$

$3x^2-3x+2x+4=5$

$-5 \quad -5$

$3x^2-x-1=0$

ok for test but better

$x=\frac{1\pm\sqrt{1-4(3)(-1)}}{6}$

$x=\frac{1\pm\sqrt{13}}{6}$

$\frac{x}{x+2}+\frac{5}{x-3}=\frac{25}{x^2-x-6}$ $(x-3)(x+2)$

$(x-3)(x+2)$

$x(x-3)+5(x+2)=25$

$x^2-3x+5x+10=25$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5$ ~~$x=3$~~ extraneous solution

b. $\frac{3}{p+6}+\frac{p}{p-6}=\frac{1}{p^2-36}$ $(p+6)(p-6)$

$(p+6)(p-6)$

$3(p-6)+p(p+6)=1$

$3p-18+p^2+6p=1$

$p^2+9p-19=0$

$p=\frac{-9\pm\sqrt{81-4(1)(-19)}}{2}$

$p=\frac{-9\pm\sqrt{157}}{2}$

c. $\frac{2x}{x+1}+\frac{4}{x+3}=\frac{x}{x^2+4x+3}$ $(x+3)(x+1)$

$(x+3)(x+1)$

$2x(x+3)+4(x+1)=x$

$2x^2+6x+4x+4=x$

$-x \quad -x$

$2x^2+9x+4=0$

$(2x+1)(x+4)=0$

$x=-\frac{1}{2}$ $x=-4$