2.6 (day 1) p. 225 2-10 (even), 11-22, 23-29 (odd)

(#2) $f(x) = \dfrac{-3}{x-1}$

$x - 1 \neq 0$

$x \neq 1$

D: $(-\infty, 1) \cup (1, \infty)$

$\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = -\infty$

(#4) $f(x) = \dfrac{2}{x^2 - 1}$

$x^2 - 1 \neq 0$

$\sqrt{x^2} \neq \sqrt{1}$

$x \neq \pm 1$

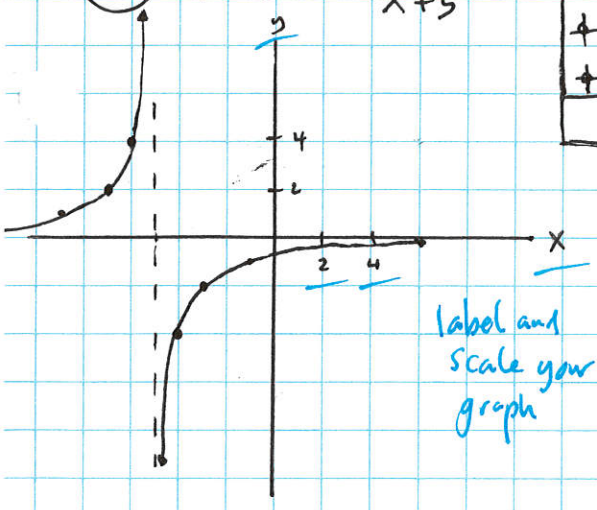
D: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$\lim_{x \to -1^-} f(x) = \infty$ $\quad \lim_{x \to 1^-} f(x) = -\infty$

$\lim_{x \to -1^+} f(x) = -\infty$ $\quad \lim_{x \to 1^+} f(x) = \infty$

pay attention to notation these are details you don't want to miss

(#6) $f(x) = -\dfrac{2}{x+5}$



label and scale your graph

- Vertical reflection
- Vertical stretch by a factor of 2
- Shift left 5 units

V.A. @ $x = -5$ H.A. @ $y = 0$

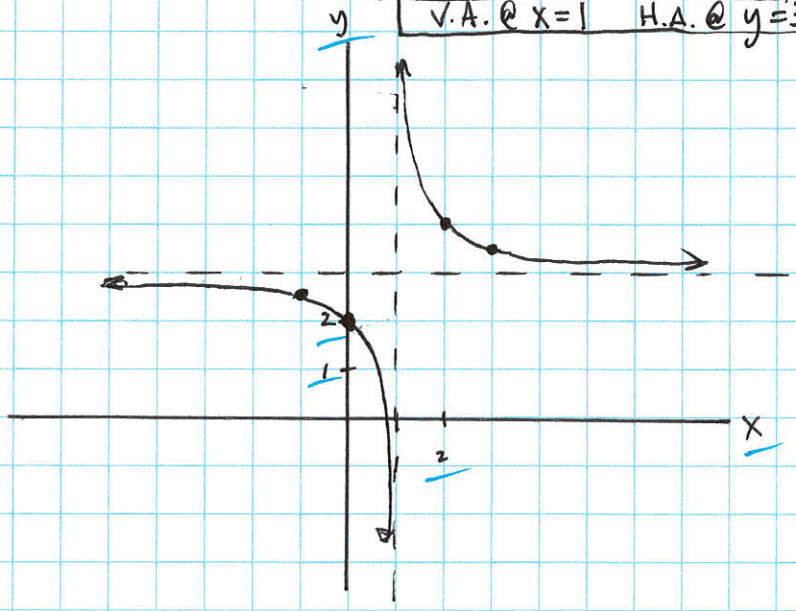
(#8) $f(x) = \dfrac{3x-2}{x-1} = 3 + \dfrac{1}{x-1}$ ← long divide to change form

$$\begin{array}{r} 3 \\ x-1 \overline{)\,3x-2} \\ -3x+3 \\ \hline 1 \end{array}$$

$f(x) = \dfrac{1}{x-1} + 3$

- Shift right 1 unit
- Shift up 3 units

V.A. @ $x = 1$ H.A. @ $y = 3$



Test question like 6, 8, 10

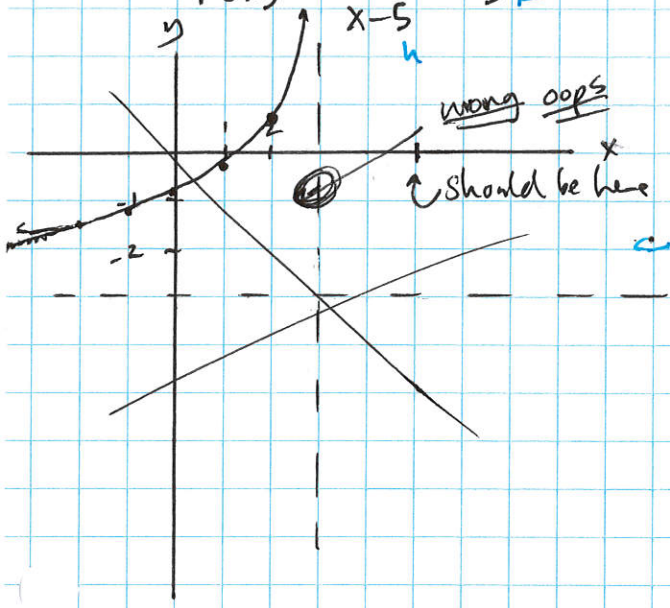
(#10) $f(x) = \dfrac{4-3x}{x-5} = -3 + \dfrac{-11}{x-5}$

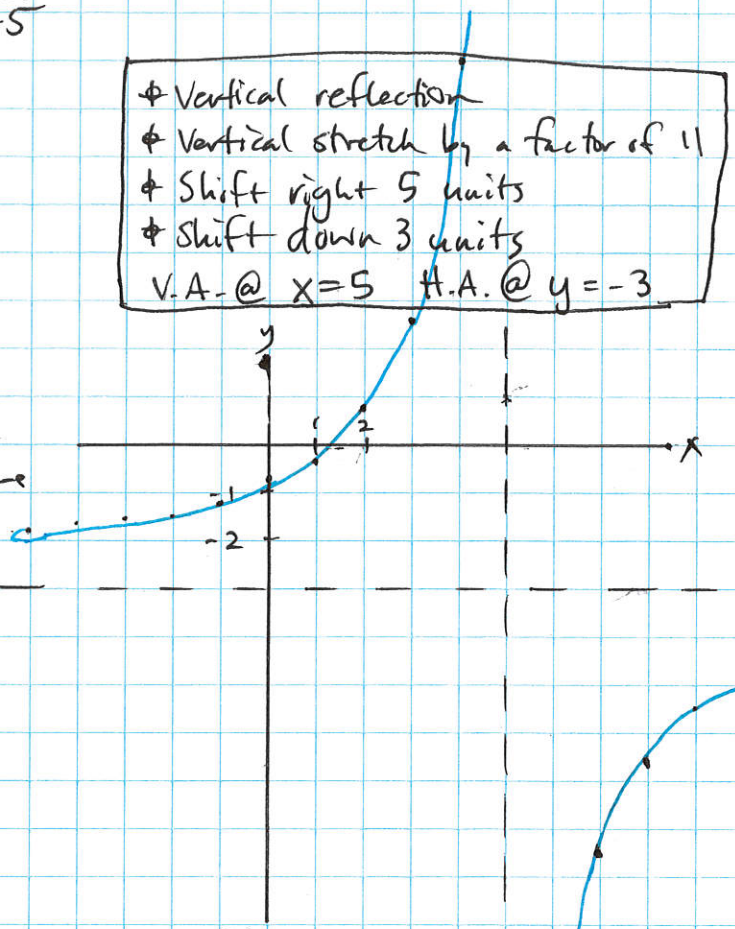
$$x-5 \overline{)-3x+4} \quad -3$$
$$+3x \mp 15$$
$$-11$$

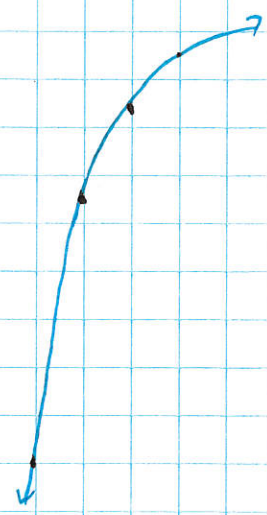
$f(x) = \dfrac{-11}{x-5} - 3$ (a, h, k)

- Vertical reflection
- Vertical stretch by a factor of 11
- Shift right 5 units
- Shift down 3 units

V.A. @ $x=5$ H.A. @ $y=-3$







(#11) $\lim_{x \to 3^-} f(x) = \infty$ (see book)

(#12) $\lim_{x \to 3^+} f(x) = -\infty$ (see book)

(#13) $\lim_{x \to \infty} f(x) = 0$ (see book)

(#14) $\lim_{x \to -\infty} f(x) = 0$ (see book)

(#15) $\lim_{x \to -3^+} f(x) = \infty$

(#16) $\lim_{x \to -3^-} f(x) = -\infty$

(#17) $\lim_{x \to -\infty} f(x) = 5$

(#18) $\lim_{x \to \infty} f(x) = 5$

#19 $f(x) = \dfrac{2x^2-1}{x^2+3}$

$x^2+3=0$

$\sqrt{x^2} \neq \sqrt{-3}$

No V.A.

degrees are =

So H.A. @ $y=2$

$\lim_{x\to-\infty} f(x) = 2 \quad \lim_{x\to\infty} f(x) = 2$

#20 $f(x) = \dfrac{3x^2}{x^2+1}$

$x^2+1=0$

$\sqrt{x^2} \neq \sqrt{-1}$

No V.A.

degrees are =

So H.A. @ $y=3$

$\lim_{x\to-\infty} f(x) = 3 \quad \lim_{x\to\infty} f(x) = 3$

#21 $f(x) = \dfrac{2x+1}{x^2-x}$

$x^2-x=0$

$x(x-1)=0$

V.A. @ $x=0 \quad x=1$

Num degree < den degree

So H.A. @ $y=0$

$\lim_{x\to-\infty} f(x) = 0 \quad \lim_{x\to\infty} f(x) = 0$

$\lim_{x\to0^-} f(x) = \infty \quad \lim_{x\to1^-} f(x) = -\infty$

$\lim_{x\to0^+} f(x) = -\infty \quad \lim_{x\to1^+} f(x) = \infty$

#22 $f(x) = \dfrac{x-3}{x^2+3x}$

$x^2+3x=0$

$x(x+3)=0$

V.A. @ $x=0 \quad x=-3$

Num degree < den degree

So H.A. @ $y=0$

$\lim_{x\to\infty} f(x) = 0 \quad \lim_{x\to\infty} f(x) = 0$

$\lim_{x\to0^-} f(x) = \infty \quad \lim_{x\to-3^-} f(x) = -\infty$

$\lim_{x\to0^+} f(x) = -\infty \quad \lim_{x\to-3^+} f(x) = \infty$

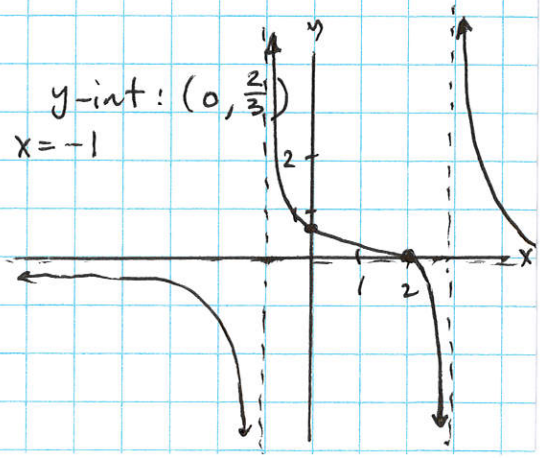
#23 $g(x) = \dfrac{x-2}{x^2-2x-3}$

$(x-3)(x+1)$

zero: $(2,0)$ y-int: $(0, \frac{2}{3})$

V.A. @ $x=3$ & $x=-1$

H.A. @ $y=0$



#25 $h(x) = \dfrac{2}{x^3 - x}$

$x(x^2 - 1) = 0$

$x = 0 \quad x^2 - 1 = 0$

$\sqrt{x^2} = \sqrt{1}$

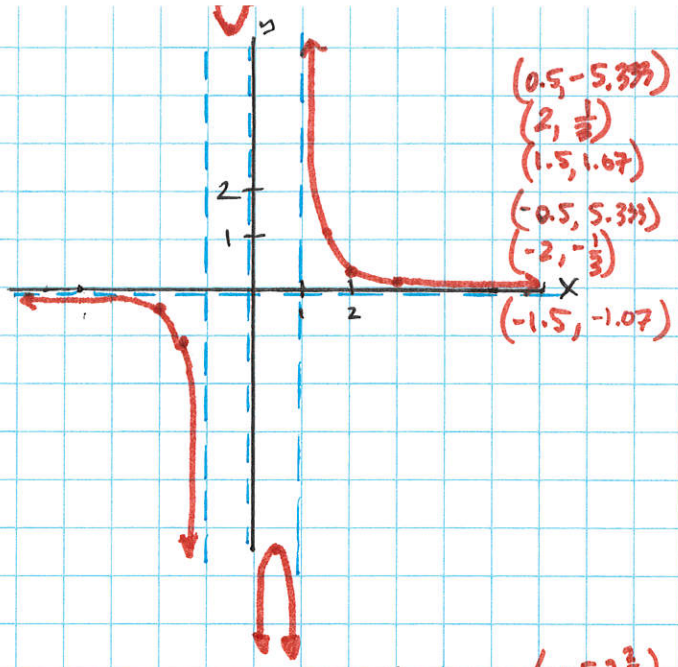
$x = \pm 1$

No x-int

No y-int

V.A @ $x = 0 \quad x = 1 \quad x = -1$

H.A. @ $y = 0$



#27 $f(x) = \dfrac{2x^2 + x - 2}{x^2 - 1}$

$2x^2 + x - 2 = 0$

~~$(2x \quad)(x \quad) = 0$~~

$x = \dfrac{-1 \pm \sqrt{1 - 4(2)(-2)}}{4}$

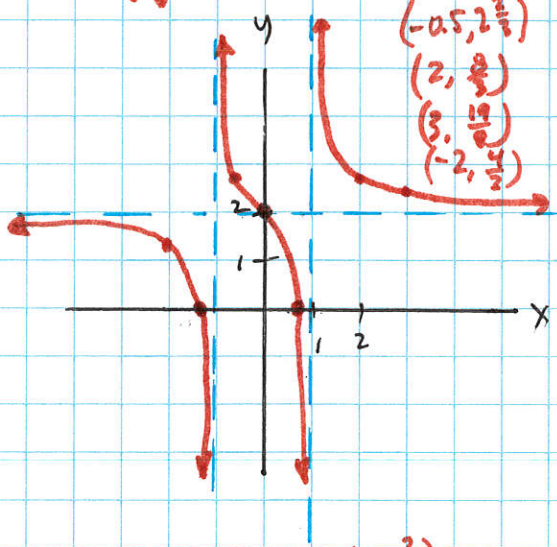
x-int: $x = \dfrac{-1 \pm \sqrt{17}}{4}$ $(.7808, 0)$ $(-1.2808, 0)$

y-int: $y = 2 \quad = (0, 2)$

V.A. @: $x^2 - 1 = 0$

$\sqrt{x^2} = \sqrt{1} \quad x = 1$ and $x = -1$

H.A. @: $y = 2$



#29 $f(x) = \dfrac{x^2 - 2x + 3}{x + 2}$

$x^2 - 2x + 3 = 0$

~~$(x - 3)(x \quad) = 0$~~

$x = 2 \pm \sqrt{4 - 4(1)(3)}$

No X-int

y-int: $(0, \frac{3}{2})$

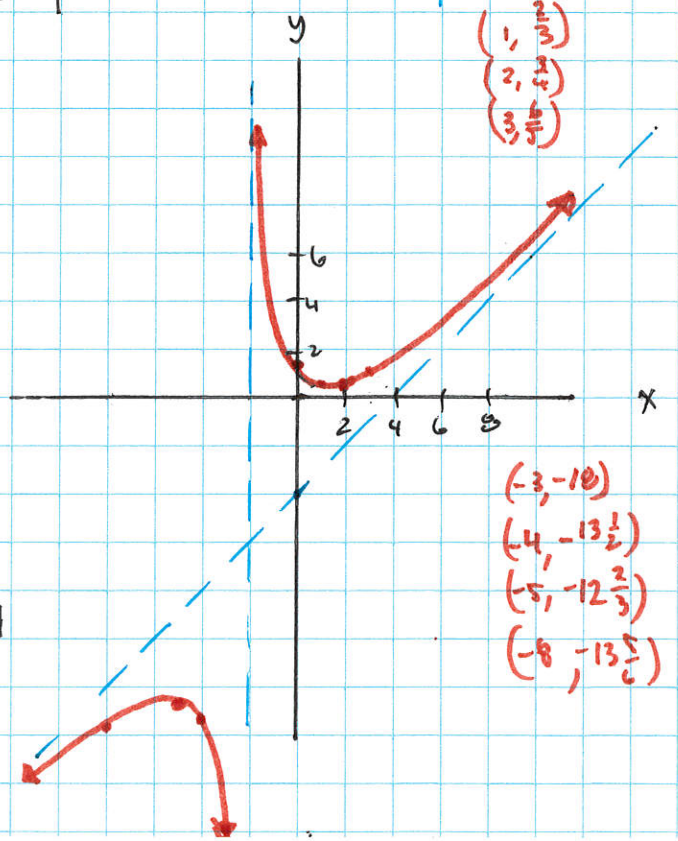
V.A.: $x + 2 = 0 \quad x = -2$

H.A.: No H.A.

but there is a slant asymptote!

$$\begin{array}{r} x - 4 \\ x+2 \overline{)\ x^2 - 2x + 3} \\ \underline{-x^2 \mp 2x} \\ -4x + 3 \\ \underline{+4x + 8} \\ 11 \end{array}$$

$y = x - 4$



2.6 (day 2) p. 226 37-43(odd), 51-60(x3), 65-68

#37 $f(x) = \dfrac{2}{2x^2 - x - 3}$

No x-int

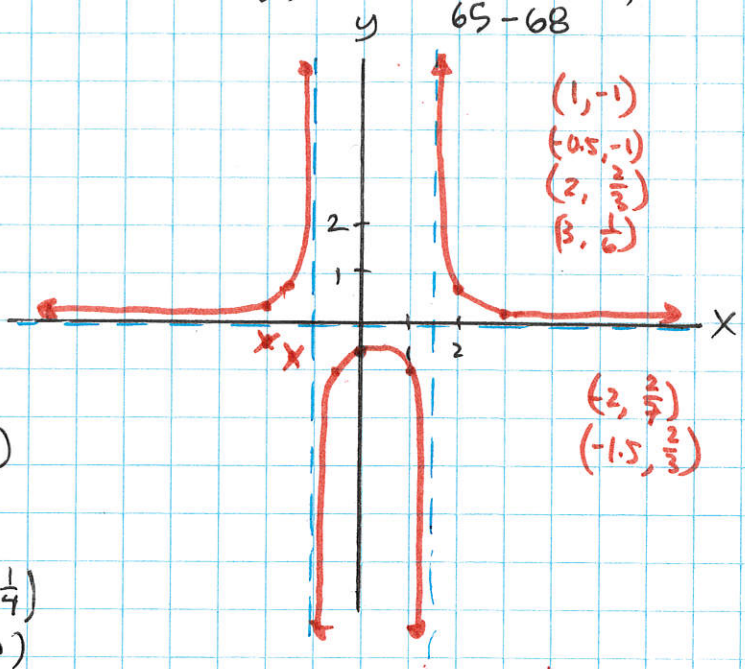
y-int: $(0, -\frac{2}{3})$

$2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$

V.A. @ $x = \frac{3}{2}$ $x = -1$

H.A. @ $y = 0$



Domain: $(-\infty, -1) \cup (-1, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

Range: $(-\infty, -\frac{16}{25}] \cup (0, \infty)$

Continuous on its domain

$f(x)$ increases on $(-\infty, -1) \cup (-1, \frac{1}{4})$

$f(x)$ decreases on $[\frac{1}{4}, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

Not bounded

Local max @ $(\frac{1}{4}, -\frac{16}{25})$

END BEHAVIOR

$\lim_{x \to -\infty} f(x) = 0$ $\lim_{x \to \infty} f(x) = 0$

Asymptote Behavior

$\lim_{x \to -1^-} f(x) = \infty$ $\lim_{x \to \frac{3}{2}^+} f(x) = \infty$

$\lim_{x \to -1^+} f(x) = -\infty$ $\lim_{x \to \frac{3}{2}^-} f(x) = -\infty$

#39 $h(x) = \dfrac{x - 1}{x^2 - x - 12}$

$x - 1 = 0$

$x = 1$

x-int: $(1, 0)$

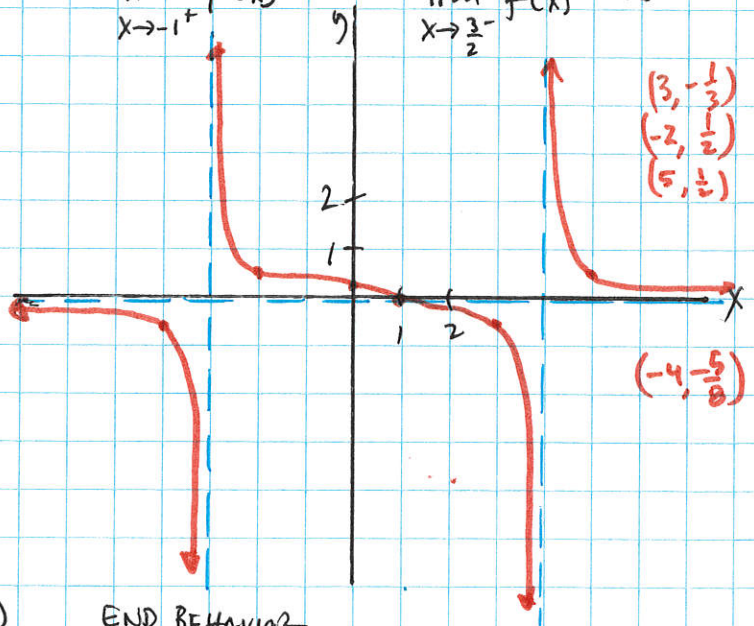
y-int: $(0, \frac{1}{12})$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

V.A. @: $x = 4$ $x = -3$

H.A. @: $y = 0$



Domain: $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$

Range: $(-\infty, \infty)$

Continuous on its domain

$h(x)$ increases on NEVER!

$h(x)$ decreases on $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$

Not bounded

No extrema

END BEHAVIOR

$\lim_{x \to -\infty} h(x) = 0$ $\lim_{x \to \infty} h(x) = 0$

A.B. $\lim_{x \to -3^-} h(x) = -\infty$ $\lim_{x \to 4^-} h(x) = -\infty$

$\lim_{x \to -3^+} h(x) = \infty$ $\lim_{x \to 4^+} h(x) = \infty$

#41 $f(x) = \dfrac{x^2+x-2}{x^2-9}$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2 \quad x=1$

x-int: $(-2,0)\ (1,0)$

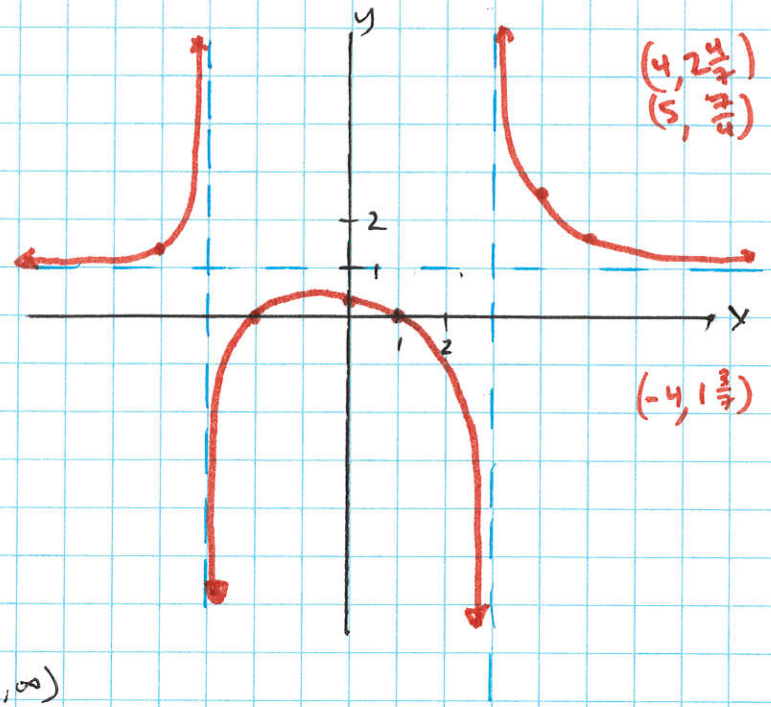
y-int: $(0, \frac{2}{9})$

$x^2-9=0$

$\sqrt{x^2}=\sqrt{9}$

V.A. @ $x=3 \quad x=-3$

H.A. @ $y=1$



Domain: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$

Range: $(-\infty, 0.260]\cup(1,\infty)$

Continuous on its domain

$f(x)$ increases on $(-\infty,-3)\cup(-3,-0.675)$

$f(x)$ decreases on $(-0.675,3)\cup(3,\infty)$

No bounds

Local max @ $(-0.675, 0.260)$

End behavior

$\lim_{x\to-\infty} f(x)=1 \quad \lim_{x\to\infty} f(x)=1$

Asymptote Behavior

$\lim_{x\to-3^-} f(x)=\infty \quad \lim_{x\to3^-} f(x)=-\infty$

$\lim_{x\to-3^+} f(x)=-\infty \quad \lim_{x\to3^+} f(x)=\infty$

#43 $h(x) = \dfrac{x^2+2x-3}{x+2}$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3 \quad x=1$

x-int: $(-3,0)\ (1,0)$

y-int: $(0,-\frac{3}{2})$

$x+2=0$

V.A. @ $x=-2$

H.A. @ No H.A.!

There is a slant thing!

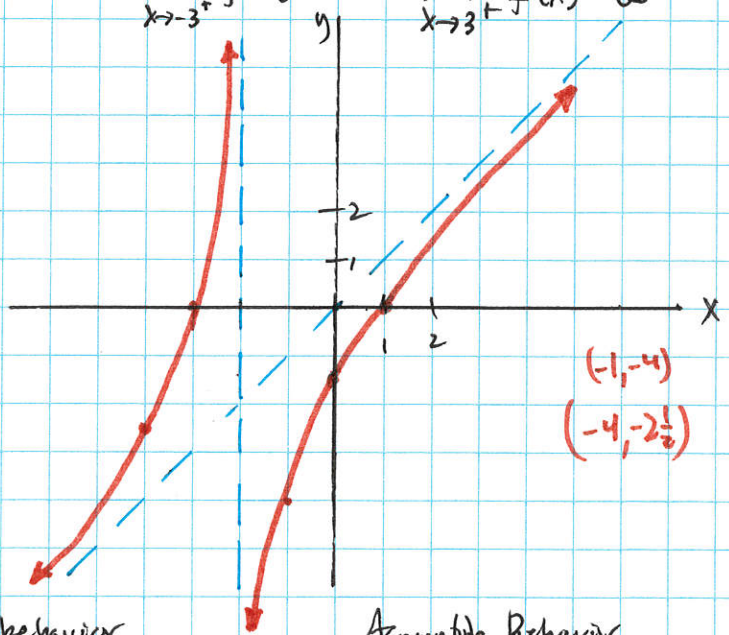
$$x+2 \overline{)\ x^2+2x-3}$$

x

$-x^2-2x$

$0-3$

$y=x$



End behavior

$\lim_{x\to-\infty} f(x)=-\infty \quad \lim_{x\to\infty} f(x)=\infty$

Asymptote Behavior

$\lim_{x\to-2^-} f(x)=\infty \quad \lim_{x\to-2^+} f(x)=-\infty$

Domain: $(-\infty,-2)\cup(-2,\infty)$

Range: $(-\infty,\infty)$

Continuous on its domain

$h(x)$ increases on $(-\infty,-2)\cup(-2,\infty)$

No bounds, no extrema,

#51 $f(x) = \dfrac{3x^2 - 2x + 4}{x^2 - 4x + 5}$

$3x^2 - 2x + 4 = 0$

$(3x \quad)(x \quad 2) = 0$

$x = 2 \pm \sqrt{4 - 4(3)(4)}$ ← Imaginary

No X-int

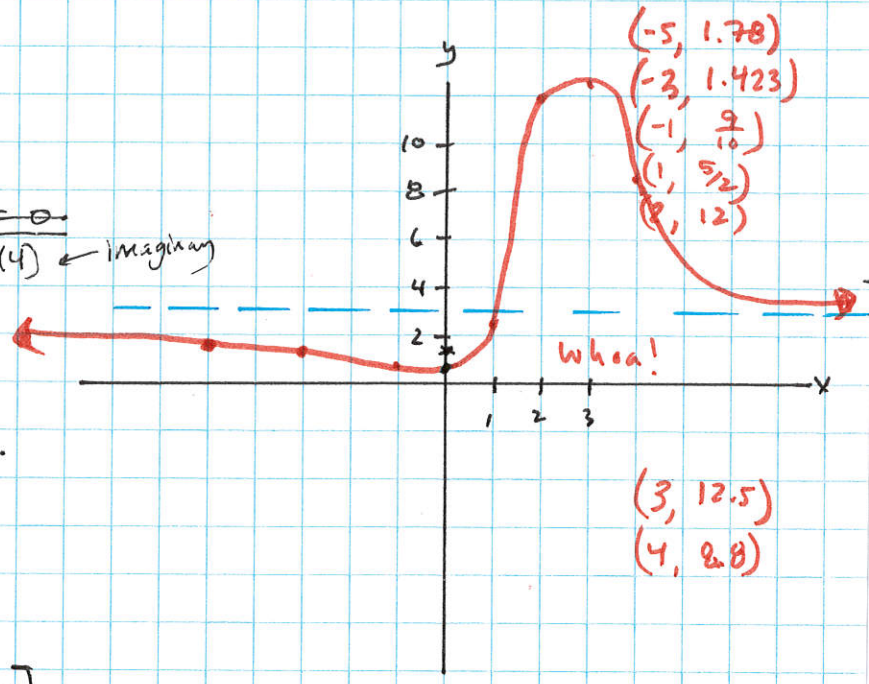
y-int: $(0, \frac{4}{5})$

$x^2 - 4x + 5 = 0$

$(x \quad 5)(x \quad 1)$

No V.A.

H.A. @ $y = 3$



Domain: $(-\infty, \infty)$

Range: $[0.773, 14.227]$

Continuous

$f(x)$ increases on $[-0.245, 2.445)$

$f(x)$ decreases on $(-\infty, -0.245) \cup (2.445, \infty)$

No symmetry, no bounds, local max $(2.445, 14.227)$

local min $(-0.245, 0.773)$

End Behavior

$\lim_{x \to -\infty} f(x) = 3$ $\lim_{x \to \infty} f(x) = 3$

#54 $k(x) = \dfrac{x^3 - 2}{x + 2}$

$x^3 - 2 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{2}$

$x = \sqrt[3]{2}$

x-int: $(\sqrt[3]{2}, 0)$

y-int: $(0, -1)$

$x + 2 = 0$

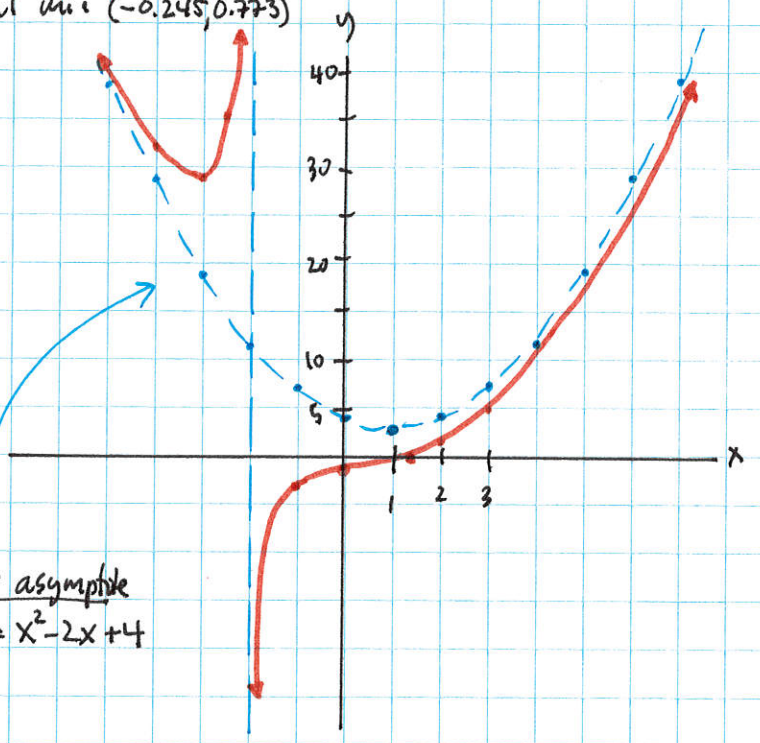
V.A. @ $x = -2$

H.A. @ No H.A.

$$\begin{array}{r} x^2 - 2x + 4 \\ x+2 \overline{)\, x^3 + 0x^2 + 0x + 2} \\ -x^3 - 2x^2 \\ \hline -2x^2 + 0x \\ +2x + 4x \\ \hline 4x - 2 \\ -4x - 8 \\ \hline -10 \end{array}$$

Slant asymptote

$y = x^2 - 2x + 4$



End behavior

$\lim_{x \to -\infty} k(x) = \infty$ $\lim_{x \to \infty} f(x) = \infty$

$k(x)$ is increasing on $(-3.104, -2) \cup (-2, \infty)$

$k(x)$ is decreasing on $(-\infty, -3.104)$

Local min $(-3.104, 28.901)$

Domain: $(-\infty, -2) \cup (-2, \infty)$

Range: $(-\infty, \infty)$, Continuous on its domain

#57 $h(x) = \frac{x^4+1}{x+1}$

Just graph it in your calculator

$x^4+1=0$

$\sqrt[4]{x^4} = \sqrt[4]{-1}$

No x-int

y-int: (0, 1)

V.A. @ $x+1=0$, $x=-1$

H.A. @ No H.A. Oblique Asymptote (or end behavior asymptote) @ $y = x^3 - x^2 + x - 1$

$$x+1 \overline{)\, x^4+0x^3+0x^2+0x+1}$$

$x^3 - x^2 + x - 1$

$-x^4 - x^3$

$-x^3+0x^2$

$+x^3+x^2$

x^2+0x

$-x^2+x$

$-x+1$

#60 $g(x) = \frac{x^5+1}{x-1}$

$x^5+1=0$

$\sqrt[5]{x^5} = \sqrt[5]{-1}$

$x=-1$

graph in calculator

x-int @ (-1, 0)

y-int @ (0, -1)

V.A. @ $x=1$

H.A. @ No H.A. Oblique Asymptote $y = x^4+x^3+x^2+x+1$

$$x-1 \overline{)\, x^5+0x^4+0x^3+0x^2+0x+1}$$

$x^4+x^3+x^2+x+1$

$-x^5+x^4$

x^4+0x^3

$-x^4+x^3$

$+x^3+0x^2$

$-x^3+x^2$

x^2+0x

$-x^2+x$

$x+1$

#66 E

$x^2+3x=0$
$x(x+3)=0$
$x\neq 0 \quad x\neq -3$

#66 A

$g(x)=\frac{2}{(x+3)}$

h value so a shift left 3 units

#67

~~A~~ There IS a V.A.
~~B~~ There IS a V.A.
~~C~~ There IS a V.A.
D There is a V.A. and Slant
~~E~~ There IS NO H.A.

#68 E $\quad \frac{x^8}{x^4}=x^{(4)}$

2.7 p. 232 1–29(odd) 34, 43, 44, 47–50

#1 $\left(\frac{x-2}{3} + \frac{x+5}{3} = \frac{1}{3}\right)3$

$x-2+x+5=1$

$2x+3=1$

$2x=-2$

$\boxed{x=-1}$

#3 $\left(x+5=\frac{14}{x}\right)x$

$x^2+5x=14$

$x^2+5x-14=0$

$(x+7)(x-2)=0$

$\boxed{x=-7}$ $\boxed{x=2}$

#5 $\left(x+\frac{4x}{x-3}=\frac{12}{x-3}\right)(x-3)$

$x(x-3)+4x=12$

$x^2-3x+4x=12$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$\boxed{x=-4}$ ~~$x=3$~~ extraneous

can't be plugged back in!

#7 $\left(x+\frac{10}{x}=7\right)x$

$x^2+10=7x$

$x^2-7x+10=0$

$(x-5)(x-2)=0$

$\boxed{x=5}$ $\boxed{x=2}$

#9 $\left(x+\frac{12}{x}=7\right)x$

$x^2+12=7x$

$x^2-7x+12=0$

$(x-4)(x-3)=0$

$\boxed{x=4}$ $\boxed{x=3}$

#11 $\left(2-\frac{1}{x+1}=\frac{1}{x^2+x}\right)x(x+1)$

$2x(x+1)-x=1$

$2x^2+2x-x=1$

$2x^2+x-1=0$

$(2x-1)(x+1)=0$

$\boxed{x=\frac{1}{2}}$ ~~$x=-1$~~ extraneous!

#13 $\left(\frac{3x}{x+5}+\frac{1}{x-2}=\frac{7}{x^2+3x-10}\right)(x+5)(x-2)$

$3x(x-2)+1(x+5)=7$

$3x^2-6x+x+5=7$

$3x^2-5x-2=0$

$(3x+1)(x-2)=0$

$\boxed{x=-\frac{1}{3}}$ ~~$x=2$~~ extraneous

#15 $\left(\frac{x-3}{x}-\frac{3}{x+1}+\frac{3}{x^2+x}=0\right)x(x+1)$

$(x-3)(x+1)-3x+3=0$

$x^2-2x-3-3x+3=0$

$x^2-5x=0$

$x(x-5)=0$

~~$x=0$~~ extraneous $\boxed{x=5}$

#17 $\left(\frac{3}{x+2}+\frac{6}{x^2+2x}=\frac{3-x}{x}\right)x(x+2)$

$3x+6=(3-x)(x+2)$

$3x+6=-x^2+x+6$

$x^2+2x=0$

$x(x+2)=0$

~~$x=0$~~ ~~$x=-2$~~ extraneous!

#19 $x=-5$ appears to be an x-int
$x=-2$ is not, its the extraneous

#21 $x=-2$ and $x=2$ both appear to be extraneous
Since the graph does not cross, but rather touches the x-axis at either

#23 $\frac{2}{x-1}+x=5$ $\quad (x-1)$

$2+x(x-1)=5(x-1)$

$2+x^2-x=5x-5$

$x^2-6x+7=0$

~~$(x-7)(x-1)=0$~~

$x=\frac{6\pm\sqrt{36-4(1)(7)}}{2}$

$x=\frac{6\pm\sqrt{8}}{2}=\frac{6\pm2\sqrt{2}}{2}=\boxed{3\pm\sqrt{2}}$

#25 $\frac{x^2-2x+1}{x+5}=0$

$x^2-2x+1=0$

$(x-1)(x-1)=0$

$\boxed{x=1}$

#27 $\frac{4x}{x+4}+\frac{5}{x-1}=\frac{15}{x^2+3x-4}$ $\quad (x+4)(x-1)$

$(x+4)(x-1)$

$4x(x-1)+5(x+4)=15$

$4x^2-4x+5x+20=15$

$4x^2+x+5=0$

~~$(4x-1)(x-5)=0$~~

$x=\frac{-1\pm\sqrt{1-4(4)(5)}}{8}$ ← $\sqrt{-}$ imaginary

$x=\frac{-1\pm\sqrt{-79}}{8}$

#29 $x^2+\frac{5}{x}=8$ $\quad x$

$x^3+5=8x$

$x^3-8x+5=0$

All decimals, use calculator.

$x=-3.1$ $\quad x=0.661$ $\quad x=2.439$

#34 $P(t) = \frac{500 + 250t}{10 + 0.5t}$

a) $P(10) = \frac{500 + 250(10)}{10 + 0.5(10)} = \frac{3000}{15} = 200$

$P(40) = 350$
$P(100) = 425$

b) Yes, degrees are the same so
H.A. @ $y = \frac{250}{0.5} = 500$

c) Can't get bigger than 500

#43 a) In calculator!

b) $E(74) = \frac{170}{74 - 58} = \frac{170}{16} = 10.625$ years ~~left~~!

#44 a) In calculator

b) about 2269 wineries

#47 D others are extraneous

#48 C

#49 E

#50 D

2.8 p. 242 2-12 (even)

#2 $f(x) = (x-7)(3x+1)(x+4)$

a) zero at $x=7$ $x=-\frac{1}{3}$ $x=-4$

b) ~~Positive over intervals~~

$f(x) > 0$ on $(-4, -\frac{1}{3}) \cup (7, \infty)$

c) $f(x) < 0$ on $(-\infty, -4) \cup (-\frac{1}{3}, 7)$

#4 $f(x) = (2x^2+5)(x-8)^2(x+1)^3$

7th degree polynomial!

$(+)(-)^2(-)^3(+)(-)^2(+)^3 \quad (+)(+)^2(+)^3$

neg -1 pos 8 pos

a) $f(x)=0$ @ $x=8$ $x=-1$

$2x^2+5=0$

$-5 \quad -5$

$\frac{2x^2}{2} = \frac{-5}{2}$

$\sqrt{x^2} \neq \sqrt{-\frac{5}{2}}$

even multiplicity kisses @ $x=8$

odd multiplicity crosses @ $x=-1$

b) $f(x) < 0$ on $(-\infty, -1)$

Not at 8 though since that is 0

c) $f(x) > 0$ on $(-1, 8) \cup (8, \infty)$

#6 $f(x) = (x+2)^3(4x^2+1)(x-9)^4$

$x=-2$ $x=\sqrt{-\frac{1}{4}}$ $x=9$

odd multiplicity even multiplicity

$(-)^3(+)(-)^4(+)^3(+)(-)^4 \quad (+)^3(+)(+)^4$

Neg -2 pos 9 pos

a) $f(x)=0$ @ $x=-2$ $x=9$

b) $f(x) < 0$ on $(-\infty, -2)$

c) $f(x) > 0$ on $(-2, 9) \cup (9, \infty)$

#8 $(2x+1)(x-2)(3x-4) \leq 0$

$x = -\frac{1}{2} \quad x = 2 \quad x = \frac{4}{3}$

$(-)(-)(-)$, $(+)(-)(-)$, $(+)(-)(+)$, $(+)(+)(+)$

Neg $-\frac{1}{2}$ pos $\frac{4}{3}$ neg 2 pos

$f(x) \leq 0$ on $(-\infty, -\frac{1}{2}] \cup [\frac{4}{3}, 2]$

#10 $(2x-7)(x^2-4x+4) > 0$

$(2x-7)(x-2)(x-2)$

$x = \frac{7}{2} \quad x = 2$

$(-)(-)(-)$, $(-)(+)(+)$, $(+)(+)(+)$

Neg 2 Neg $\frac{7}{2}$ pos

$f(x) > 0$ on $(\frac{7}{2}, \infty)$

#12 $x^3 - 4x^2 + x + 6 \leq 0$

3 | 1 −4 1 6

3 −3 −6

1 −1 −2 | 0

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$(-)(-)(-)$, $(-)(-)(+)$, $(-)(+)(+)$, $(+)(+)(+)$

neg −1 pos 2 Neg 3 pos

$f(x) = (x-3)(x-2)(x+1)$

$f(x) \leq 0$ on $(-\infty, -1] \cup [2, 3]$

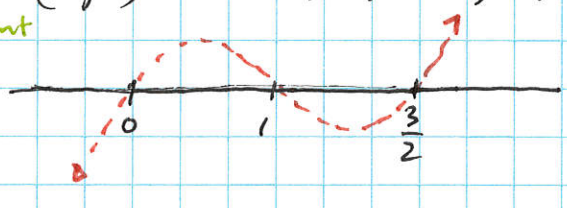
2.8 (deg 2) 14-20 even 25-27, 33, 36, 37, 40, 59

#14 $2x^3 - 5x^2 + 3x < 0$ ← for just a moment

$x(2x^2 - 5x + 3) = 0$

$(x)(2x-3)(x-1)$

$x = 0 \quad x = \frac{3}{2} \quad x = 1$



$f(x) < 0$ on $(-\infty, 0) \cup (1, \frac{3}{2})$

#16 $x^3 - 4x^2 - x + 4 \leq 0$ ← factoring by grouping

$x^2(x-4) - 1(x-4) = 0$

$(x-4)(x^2-1) = 0$

$x = 4 \quad x^2 - 1 = 0$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

$f(x) \leq 0$ on $(-\infty, -1] \cup [1, 4]$

#18 $-x^3-3x^2-9x+4<0$

-1	-1	-3	-9	4
		1	2	7
	-1	-2	7	11

NONE WORK

USE CALCULATOR

pos .388 neg

$f(x)<0$ on $(.388, \infty)$

#20 $3x^4-5x^3-12x^2+12x+16\geq 0$

-1	3	-5	-12	+12	16
		-3	8	4	-16
	3	-8	-4	16	0
2		6	-4	-16	
	3	-2	-8	0	
2		6	8		
	3	4	0		

$3x+4=0$

$x=-\frac{4}{3}$

$(-)(-)^2(-)$ $(-)(-)^2(+)$ $(+)(-)^2(+)$ $(+)(+)^2(+)$

pos $-\frac{4}{3}$ neg -1 pos 2 pos

$f(x)=(x+1)(x-2)^2(3x+4)$

$f(x)\geq 0$ $(-\infty, -\frac{4}{3}]\cup[-1,\infty)$

#25 $f(x)=\frac{x-1}{(2x+3)(x-4)}$

a) x-int: $x-1=0$

$x=1$

V.A.: $2x+3=0$ $x-4=0$

b) $x=-\frac{3}{2}$ $x=4$

$\frac{(-)}{(-)(-)}$ V.A. $\frac{(-)}{(+)(-)}$ zero $\frac{(+)}{(+)(-)}$ V.A. $\frac{(+)}{(+)(+)}$

Neg $-\frac{3}{2}$ pos, Neg 4 pos

c) $f(x)>0$ on $(-\frac{3}{2}, 1)\cup(4,\infty)$

d) $f(x)<0$ on $(-\infty, -\frac{3}{2})\cup(1,4)$

#26 $f(x)=\frac{(2x-7)(x+1)}{(x+5)}$

a) x-int: $x=\frac{7}{2}$ $x=-1$

b) V.A.: $x=-5$

$\frac{(-)(-)}{(-)}$ V.A. $\frac{(-)(-)}{(+)}$ zero $\frac{(-)(+)}{+}$ zero $\frac{(+)(+)}{(+)}$

Neg -5 pos -1 neg $\frac{7}{2}$ pos

c) $f(x)>0$ on $(-5,-1)\cup(\frac{7}{2},\infty)$

d) $f(x)<0$ on $(-\infty,-5)\cup(-1,\frac{7}{2})$

#27 $f(x) = x\sqrt{x+3}$

a) $x = 0 \quad x = -3$

b) $x + 3 \geq 0$ undefined $x < -3$

$x \geq -3$

c) $f(x) > 0$ on $(0, \infty)$

d) $f(x) < 0$ on $(-3, 0)$

(-)(und), (-)(+), (+)(+)

undefind −3 neg 0 pos

#33 $\dfrac{x-1}{x^2-4} < 0$

$(x+2)(x-2)$

$x - 1 = 0 \quad x^2 - 4 = 0$

$x = 1 \quad \sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$\dfrac{(-)}{(+)}$ V.A. $\dfrac{(-)}{(-)}$ zero $\dfrac{(+)}{(-)}$ V.A. $\dfrac{(+)}{(+)}$

Neg −2 pos, Neg 2 pos

$f(x) < 0$ on $(-\infty, -2) \cup (1, 2)$

#36 $\dfrac{x^2-4}{x^2+4} > 0$

$(x+2)(x-2) = 0$

$x = \pm 2$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \sqrt{-4}$

$\dfrac{(+)}{(+)}$ V.A. $x = -2$ $\dfrac{(-)}{(+)}$ V.A. $x = 2$ $\dfrac{(+)}{(+)}$

pos −2 neg 2 pos

$f(x) > 0$ on $(-\infty, -2) \cup (2, \infty)$

#37 $\dfrac{(x+4)(x-3)}{(x-2)(x-2)}$ $\dfrac{x^2+x-12}{x^2-4x+4} > 0$

x-int: $x = -4 \quad x = 3$

V.A. $x = 2$

$\dfrac{(-)(-)}{(-)(-)}$ V.A. $x = -4$ $\dfrac{(+)(-)}{(-)(-)}$ V.A. $x = 2$ $\dfrac{(+)(-)}{(+)(+)}$ V.A. $x = 3$ $\dfrac{(+)(+)}{(+)(+)}$

pos −4 Neg 2 Neg 3 pos

$f(x) > 0$ on $(-\infty, -4) \cup (3, \infty)$

#40 $\dfrac{x^3-4x}{x^2+2} \leq 0$

$x(x^2-4) = 0$

$(x)(x+2)(x-2) = 0$

$x = 0 \quad x = -2 \quad x = 2$

$x^2 + 2 = 0$

Nope

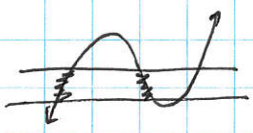
$\dfrac{(-)(-)(-)}{(+)}$ V.A. $x = -2$ $\dfrac{(-)(+)(-)}{(+)}$ V.A. $x = 0$ $\dfrac{(+)(+)(-)}{(+)}$ V.A. $x = 2$ $\dfrac{(+)(+)(+)}{(+)}$

Neg −2 pos 0 Neg 2 pos

$f(x) \leq 0$ on $(-\infty, -2] \cup [0, 2]$

#59 $(x)(15-2x)(12-2x) \leq 100\ in^3$

graph out



$f(x) \leq 100\ in^3$ on $(0, .69) \cup (4.2, 6)$