Practice Test: Chapter 1.4-1.7 Name: ____________________

Pre-Calculus | Mr. Cooper **(Non-Calculator)** **Date:**

1. Let $f(x) = \frac{1}{x-1}$ and let $g(x) = \sqrt{2x}$. Find an expression and give the domain for each of the following.

a. $(f+g) = \frac{1}{x-1} + \sqrt{2x}$

$2x \geq 0$ $\quad x-1 \neq 0$

$x \geq 0$ $\quad x \neq 1$

$\boxed{D: [0,1)\cup(1,\infty)}$

b. $(fg) = \frac{\sqrt{2x}}{x-1}$

$2x \geq 0$ $\quad x-1 \neq 0$

$x \geq 0$ $\quad x \neq 1$

$\boxed{D: [0,1)\cup(1,\infty)}$

c. $f(g(x)) = \frac{1}{\sqrt{2x}-1}$

Radical must be ≥ 0

denominator can't equal 0

$\sqrt{2x} \geq 0$ $\quad \sqrt{2x}-1 \neq 0$

$2x \geq 0$ $\quad \sqrt{2x} \neq 1$

$x \geq 0$ $\quad 2x \neq 1$

$x \neq \frac{1}{2}$

$\boxed{D: [0,\tfrac{1}{2})\cup(\tfrac{1}{2},\infty)}$

d. $(g \circ f)(x) = \sqrt{2\left(\frac{1}{x-1}\right)} = \sqrt{\frac{2}{x-1}}$

$\frac{2}{x-1} \geq 0$

$\frac{+}{+}$ only way for this fraction to be > 0 since numerator is already positive!

So $x-1 > 0$

$x > 1$

No equal since x-1 is in the denominator

$\boxed{D: (1,\infty)}$

2. Let $f(x) = \sqrt{2x-1}$ and let $g(x) = \frac{1}{x} + 1$. Find an expression and give the domain for each of the following.

a. $(g-f) = \frac{1}{x} + 1 - \sqrt{2x-1}$

$2x-1 \geq 0$ $\quad x \neq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

$\boxed{D: [\tfrac{1}{2},\infty)}$

Already outside domain

b. $g(f(x)) = \frac{1}{\sqrt{2x-1}} + 1$

$2x-1 > 0$

$2x > 1$

$x > \frac{1}{2}$

$\boxed{D: (\tfrac{1}{2},\infty)}$

c. $(f \circ g)(x) = \sqrt{2\left(\frac{1}{x}+1\right) - 1}$

$= \sqrt{\frac{2}{x} + 2 - 1} = \sqrt{\frac{2}{x}+1}$

$x \neq 0$

$\frac{2}{x} + 1 \geq 0$

$\frac{2}{x} \geq -1$

2 is positive so think

$\frac{+}{+} \geq -1$; $\frac{2}{x>0} \geq -1$

$\frac{+}{\text{some}(-)} \geq -1$; $\frac{2}{x \leq -2} \geq -1$

⊛ careful! Tough one.

$\boxed{D: (-\infty,-2]\cup(0,\infty)}$

d. $\frac{f}{g} = \frac{\sqrt{2x-1}}{\frac{1}{x}+1}$

$2x-1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

$\boxed{D: [\tfrac{1}{2},\infty)}$

$x \neq 0$

$\frac{1}{x} + 1 \neq 0$

$\frac{1}{x} \neq -1$

$x \neq -1$

Both already outside domain from radical

3. Let $f(x) = (x-1)^2$ and $g(x) = \sqrt{x+5}$

a. Find $\frac{f}{g}$ and give the domain

$\frac{f}{g} = \frac{(x-1)^2}{\sqrt{x+5}}$

$x+5 \geq 0$

in denominator

$x > -5$

$\boxed{D: (-5,\infty)}$

b. Find $\frac{g}{f}$ and give the domain

$\frac{g}{f} = \frac{\sqrt{x+5}}{(x-1)^2}$

$x+5 \geq 0$ $\quad x-1 \neq 0$

$x \geq -5$ $\quad x \neq -1$

$\boxed{D: [-5,1)\cup(1,\infty)}$

4. Let $f(x) = x^2$ and $g(x) = \sqrt{x+1}$.

a. Find $\frac{f}{g}$ and give the domain

$\frac{f}{g} = \frac{x^2}{\sqrt{x+1}}$

$x+1 > 0$

$x > -1$

$\boxed{D: (-1, \infty)}$

b. Find $\frac{g}{f}$ and give the domain

$\frac{g}{f} = \frac{\sqrt{x+1}}{x^2}$

$x+1 \geq 0$ $\quad$ $x^2 \neq 0$

$x \geq -1$ $\quad$ $x \neq 0$

$\boxed{D: [-1, 0) \cup (0, \infty)}$

5. Find a formula for $f^{-1}(x)$

a. $f(x) = \frac{2x}{5-x}$

$(5-y)x = \frac{2y}{\cancel{(5-y)}}\cancel{(5-y)}$

$5x - xy = 2y$

$\quad + xy \quad\quad +xy$

$5x = 2y + xy$

$(2+x)\quad \frac{5x}{(2+x)} = \frac{y(2+x)}{(2+x)}$

$\boxed{f^{-1}(x) = \frac{5x}{2+x}}$

b. $P(x) = \frac{7}{x-3}$

$x = \frac{7}{y-3}$

$7 \cdot \frac{1}{x} = \frac{y-3}{7} \cdot 7$

$\frac{7}{x} = y - 3$

$+3 \quad\quad +3$

$\boxed{f^{-1}(x) = \frac{7}{x} + 3}$

c. $h(x) = \frac{1+x}{2x-3}$

$(2y-3)x = \frac{1+y}{\cancel{(2y-3)}}\cancel{(2y-3)}$

$2xy - 3x = 1 + y$

$-2xy \quad -1 \quad -1 \quad -2xy$

$-3x - 1 = y - 2xy$

$-3x - 1 = y(1-2x)$

$\boxed{\frac{-3x-1}{1-2x} = f^{-1}(x)}$

d. $g(x) = 3x - 2$

$x = 3y - 2$

$+2 \quad\quad +2$

$\frac{x+2}{3} = \frac{3y}{3}$

$\boxed{f^{-1}(x) = \frac{x+2}{3}}$

6. Decompose the following functions

a. $f(x) = \frac{2}{(x^2 - 5x)}$

$f(x) = h(g(x))$

$h(x) = \frac{2}{(x)}$ $\quad$ $g(x) = (x^2 - 5x)$

b. $g(x) = (3x^2 + 5x - 7)^3$

$g(x) = h(f(x))$

$h(x) = (x)^3$ $\quad$ $f(x) = (3x^2 + 5x - 7)$

7. Describe in detail the sequence of transformations off the parent function used to graph these equations.

a. $y = 2(x-4)^2 - 5$

- vertical stretch by a factor of 2
- Shift right 4 units
- Shift down 5 units

b. $P(x) = -\frac{1}{2}|x+4| + 3$

- vertical reflection
- vertical shrink by a factor of $\frac{1}{2}$
- shift left 4 units
- Shift up 3 units

c. $f(x) = \frac{7}{9}\sqrt{x-1} + 1$

- vertical shrink by a factor of $\frac{7}{9}$
- shift right 1 unit
- shift ~~down~~ up 1 unit

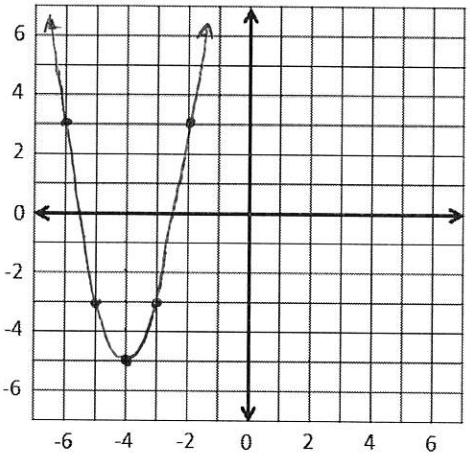
d. $h(t) = -\frac{4}{3}(t+2)^3 + \frac{1}{3}$

- vertical reflection
- vertical stretch by a factor of $\frac{4}{3}$
- shift left 2 units
- shift up $\frac{1}{3}$ units

8. In the following, a function is given as well as transformations made to the graph. Write the equation whose graph is $g(x)$. Then graph $g(x)$.

a. $y = x^2$: a shift left 4 units, a vertical stretch by a factor of 2, then then a shift down 5 units.

$f(x) = 2(x+4)^2 - 5$

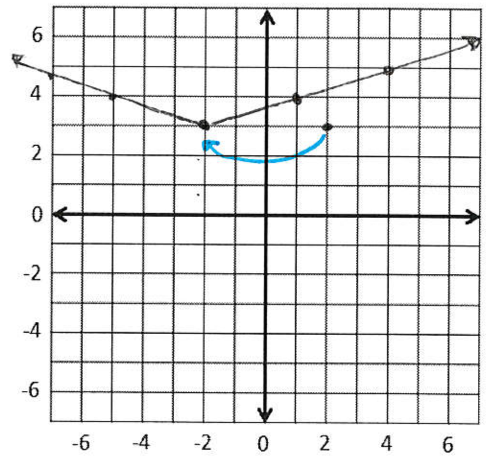


b. $y = |x|$: a shift right of 2 units, a vertical shrink by a factor of 3, a reflection across the y-axis, then a shift up 3 units

make x −

$f(x) = \frac{1}{3}|-x-2| + 3$

$|-(x-2)|$ would reflect about vertex



9. Optimization **(CALCULATOR OK)** –be able to do 2 of the 3

a. Dracula is trying to market a new soft drink called "Red Drink". Because of your mad optimizing skillz he has decided to come to you to design a cylindrical can that would minimize the amount of material used. The "Red Drink" must contain 150 cubic inches of drink. What are the dimensions of the can that uses the least amount of material?

$V = \pi r^2 h$

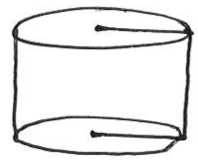
$\frac{150}{\pi r^2} = \frac{\cancel{\pi r^2} h}{\cancel{\pi r^2}}$

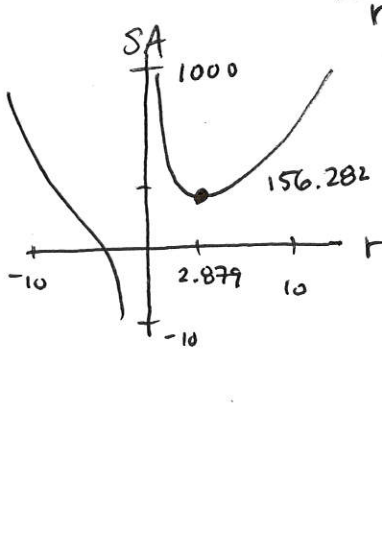
$\frac{150}{\pi r^2} = h$

$SA = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\cancel{\pi r}\left(\frac{150}{\pi r^{\cancel{2}}}\right)$

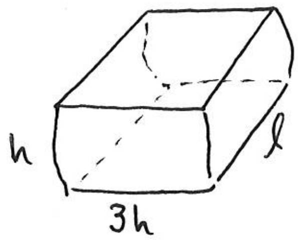
$SA = 2\pi r^2 + \frac{300}{r}$





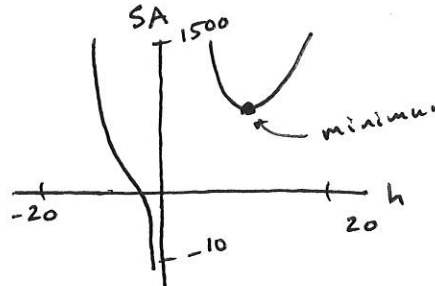
The radius would need to be 2.879 inches, height would be 5.76 inches for a minimized surface area of 156.282 inches^2.

b. A company makes a block of cheese designed to have a width triple that of the height. If they demand the block to be $1500mm^3$, what would the dimensions of the block be that would minimize the exposed surface area?



$$\frac{1500}{3h^2} = \frac{(h)(3h)(l)}{3h^2}$$

$$l = \frac{1500}{3h^2} = \frac{500}{h^2}$$



$$SA = 2(3h)(h) + 2\left(\frac{500}{h^2}\right)(h) + 2(3h)\left(\frac{500}{h^2}\right)$$

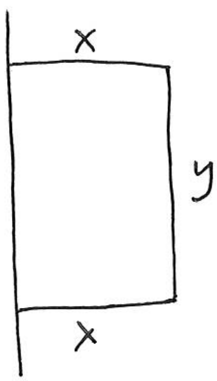
$$= 6h^2 + \frac{1000}{h} + \frac{3000}{h}$$

$$SA = 6h^2 + \frac{4000}{h}$$

Dimensions of block
$h = 6.934\ mm$
$w = 20.802\ mm$
$l = 10.399\ mm$

Minimum $SA = 865.35\ mm^2$

c. Farmer John has a beautiful daughter. To win over Farmer John, and impress his daughter, you want to help him build a fence that would maximize the area enclosed for his goats. Farmer John has $1{,}000ft$ of fence, and you plan on using one side of his house as part of the fence. What are the dimensions for the fencing?



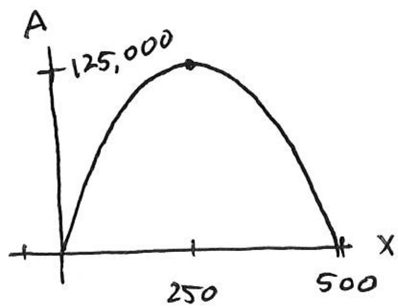
$$2x + y = 1000$$

$$y = 1000 - 2x$$

$$A = xy$$

$$A = x(1000 - 2x)$$

$$A = 1000x - 2x^2$$



250 feet by 500 feet will maximize the area at 125,000 feet^2.