Chapter 5.1-5.4 Practice Test

Pre-Calculus | Cooper

Name _____

Date _____

Period _____

Grade _____

Prove the identities.

1. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$\frac{\sin\theta}{\sin\theta}\ \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\ \frac{\cos\theta}{\cos\theta} =$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} =$

$\frac{1}{\sin\theta\cos\theta} =$

$\boxed{\sec\theta\csc\theta = \sec\theta\csc\theta}$

2. $\csc x - \cos x\cot x = \sin x$

$\frac{1}{\sin x} - \frac{\cos x}{1}\frac{\cos x}{\sin x} =$

$\frac{1-\cos^2 x}{\sin x} =$

$\frac{\sin^{\cancel{2}} x}{\cancel{\sin x}} =$

$\boxed{\sin x = \sin x}$

3. $\cos x\,[\tan x + \sin x\cot x] = \sin x + \cos^2 x$

$\cos x\left(\frac{\sin x}{\cos x} + \frac{\sin x\cos x}{\sin x}\right) =$

$\sin x + \frac{\cancel{\sin x}\cos^2 x}{\cancel{\sin x}} =$

$\boxed{\sin x + \cos^2 x = \sin x\cos^2 x}$

4. $\frac{\cos^2 x - 1}{\cos x} = -\tan x\sin x$

$\frac{\cancel{1} - \sin^2 x - \cancel{1}}{\cos x} =$

$\frac{-\sin x\sin x}{\cos x} =$

$\boxed{-\tan x\sin x = -\tan x\sin x}$

5. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$

$\cancel{(\cos^2 x + \sin^2 x)}(\cos^2 x - \sin^2 x)$

$\boxed{\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x}$

6. $\frac{\cos\theta}{1+\sin\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$

$\frac{1-\sin\theta}{1-\sin\theta}\ \frac{1+\sin\theta}{1+\sin\theta}$

$\frac{\cos\theta - \cancel{\sin\theta\cos\theta} + \cos\theta + \cancel{\sin\theta\cos\theta}}{1-\sin^2\theta} =$

$\frac{2\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta} =$

$\frac{2}{\cos\theta} =$ $\boxed{2\sec\theta = 2\sec\theta}$

7. $\frac{(1-\cos x)(1+\cos x)}{\cos^2 x} = \tan^2 x$

$\frac{1-\cos^2 x}{\cos^2 x} =$

$\frac{\sin^2 x}{\cos^2 x} =$

$\boxed{\tan^2 x = \tan^2 x}$

8. $\tan^2\theta - \sin^2\theta = \sin^2\theta\tan^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{1}\frac{\cos^2\theta}{\cos^2\theta} =$

$\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta} =$

$\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} =$

$\boxed{\tan^2\theta\sin^2\theta = \tan^2\theta\sin^2\theta}$

9. $\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right)=\cos x$

$\cos\frac{\pi}{3}\cos x - \cancel{\sin\frac{\pi}{3}\sin x} + \cos\frac{\pi}{3}\cos x + \cancel{\sin\frac{\pi}{3}\sin x} =$

$2\cos\frac{\pi}{3}\cos x =$

$2\left(\frac{1}{2}\right)\cos x =$

$\boxed{\cos x = \cos x}$

10. $\sin x\cos(x+y)-\cos x\sin(x+y)=-\sin y$

$\sin x(\cos x\cos y - \sin x\sin y) - \cos x(\sin x\cos y + \cos x\sin y)$

$\cancel{\sin x\cos x\cos y} - \sin^2 x\sin y - \cancel{\cos x\sin x\cos y} - \cos^2 x\sin y =$

$-\sin^2 x\sin y - \cos^2 x\sin y =$

$-\sin y(\sin^2 x + \cos^2 x) =$

$\boxed{-\sin y = -\sin y}$

11. $\cos(\frac{\pi}{4}+x)+\cos(\frac{\pi}{4}-x)=\sqrt{2}\cos x$

$\cos\frac{\pi}{4}\cos x - \cancel{\sin\frac{\pi}{4}\sin x} + \cos\frac{\pi}{4}\cos x + \cancel{\sin\frac{\pi}{4}\sin x} =$

$2\cos\frac{\pi}{4}\cos x =$

$\cancel{2}\frac{\sqrt{2}}{\cancel{2}}\cos x =$

$\boxed{\sqrt{2}\cos x = \sqrt{2}\cos x}$

12. $\frac{\cos 2x-1}{\sin 2x}=-\tan x$

$\frac{\cancel{1}-2\sin^2 x-\cancel{1}}{2\sin x\cos x} =$

$\frac{-\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin x}\cos x} =$

$-\frac{\sin x}{\cos x} =$

$\boxed{-\tan x = -\tan x}$

13. $\cot(\alpha)=\frac{1+\cos(2\alpha)}{\sin(2\alpha)}$

$= \frac{\cancel{1}+2\cos^2\alpha-\cancel{1}}{2\sin\alpha\cos\alpha}$

$= \frac{\cancel{2}\cos^{\cancel{2}}\alpha}{\cancel{2}\sin\alpha\cancel{\cos\alpha}}$

$= \frac{\cos\alpha}{\sin\alpha}$

$\boxed{\cot\alpha = \cot\alpha}$

Solve the following equations. Let's restrict the domain to $(0 \le \theta < 2\pi)$ or $(0 \le x < 2\pi)$.

14. $2\sin 2\theta = 2\cos\theta$

$2(\sin 2\theta) - 2\cos\theta = 0$

$2(2\sin\theta\cos\theta) - 2\cos\theta = 0$

$2\cos\theta(2\sin\theta - 1) = 0$

$2\cos\theta = 0 \qquad 2\sin\theta - 1 = 0$

$\cos\theta = 0 \qquad \sin\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$

15. $\sin 2x - \cot x = 0$

$2\sin x\cos x - \frac{\cos x}{\sin x} = 0$

$\cos x\left(2\sin x - \frac{1}{\sin x}\right) = 0$

$\cos x = 0 \qquad \sin x \cdot 2\sin x = \frac{1}{\sin x} \cdot \sin x \qquad \sin x = \pm\frac{1}{\sqrt{2}}$

$2\sin^2 x = 1$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

16. $2\cos^2 x + \cos x = \cos 2x$

$2\cos^2 x + \cos x - \cos 2x = 0$

$2\cos^2 x + \cos x - (2\cos^2 x - 1) = 0$

$\cancel{2\cos^2 x} + \cos x - \cancel{2\cos^2 x} + 1 = 0$

$\cos x + 1 = 0$

$\cos x = -1$

$\boxed{x = \pi}$

17. $\cos\theta\cot\theta = \cos\theta$

$\cos\theta\frac{\cos\theta}{\sin\theta} - \cos\theta = 0$

$\cos\theta(\cot\theta - 1) = 0$

$\cos\theta = 0 \qquad \cot\theta = 1$

$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}}$

18. $2\sin^2 x - 3\sin x = 2$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$2\sin x = -1 \qquad \cancel{\sin x = 2}$

$\sin x = -\frac{1}{2}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

19. $\cos 2\theta = \cos^2\theta + \sin\theta - 2$

$0 = -\cos 2\theta + \cos^2\theta + \sin\theta - 2$

$= -(1 - 2\sin^2\theta) + (1 - \sin^2\theta) + \sin\theta - 2$

$= -1 + \cancel{2\sin^2\theta} + 1 - \cancel{\sin^2\theta} + \sin\theta - 2$

$= \sin^2\theta + \sin\theta - 2$

$= (\sin\theta + 2)(\sin\theta - 1)$

$\cancel{\sin\theta = -2} \qquad \sin\theta = 1$

$\boxed{\theta = \frac{\pi}{2}}$

20. $\frac{\cos^3 x}{\sin x} = \cot x$

$\frac{\cos^3 x}{\sin x} - \cot x = 0$

$\frac{\cos^3 x}{\sin x} - \frac{\cos x}{\sin x} =$

$\frac{\cos^3 x - \cos x}{\sin x} =$

$\cos x(\cos^2 x - 1) = 0$

$\cos x = 0 \qquad \sqrt{\cos^2 x} = \sqrt{1}$

$\cancel{\cos x = \pm 1}$ extraneous

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2},}$

21. $\sin x - \sin^2 x = 3\cos^2 x$

$\sin x - \sin^2 x - 3\cos^2 x = 0$

$\sin x - \sin^2 x - 3(1 - \sin^2 x) = 0$

$\sin x - \cancel{\sin^2 x} - 3 + 3\cancel{\sin^2 x} = 0$

$2\sin^2 x + \sin x - 3 = 0$

$(2\sin x + 3)(\sin x - 1) = 0$

$\frac{2\sin x}{2} = \frac{-3}{2} \qquad \sin x = 1$

$\cancel{\sin x = -\frac{3}{2}}$

$\boxed{x = \frac{\pi}{2}}$

22. Find the exact value of $\cos 15^\circ$ (use a formula!).

$\cos(45-30) =$

$\cos 45 \cos 30 + \sin 45 \sin 30 =$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

23. Find the exact value of $\sin 105^\circ$ (use a formula!).

$\sin(60+45) =$

$\sin 60 \cos 45 + \cos 60 \sin 45 =$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

24. Find the exact value of $\cos 165^\circ$ (use a formula!).

$\cos(120+45) =$

$\cos 120 \cos 45 - \sin 120 \sin 45$

$\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$